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THE SUBJECT MATTER OF FORMAL LOGIC[1]

IF by logic is meant a clear, accurate, and orderly intellectual procedure, then the subject of logic, as presented in current text-books comes near being the most illogical in our chaotic curriculum. Defined almost universally as dealing with the laws of thought and devoting considerable space to the way in which the mind proceeds or fails to proceed in conception, judgment, and reasoning, it is neither clearly distinguished from psychology nor frankly treated as a branch of that modern universal science. In addition, the following miscellany is interjected into our science, designed to train young people in the habits of clear thinking: (1) Linguistic information as to the meaning and use of words, extending often to their history, and grammatical considerations as to the structure of sentences; (2) rhetorical considerations as to the persuasive force of various arguments; (3) metaphysical considerations as to the reality or unreality of universals and particulars and their relations; (4) epistemologic, *i. e.*, mixed psychologic and metaphysical, considerations as to the nature of knowledge and its relation to what is called the world of reality; (5) catalogues of miscellaneous ancient errors, under the head of material fallacies; (6) pedagogic directions as to the conduct of the human understanding, teaching us how to discover the cause of typhoid or of some other disease of which the cause is already known; (7) miscellaneous general considerations of various other sciences and their histories, which pretend to describe the essence of scientific method; and (8) the rudiments of formal or symbolic logic, as in the theory of classes or syllogisms, which, as developed in such books as Couturat's "Algebra of Logic," is strictly a mathematical science, though it need not necessarily be expressed in special symbols.

Mixed studies, like mixed races and mixed constitutions, show the greatest vitality, and there could be no valid objection to the same text-book treating all these important matters, provided the information given were accurate and the various points of view

[1] Read before the American Philosophical Association, December, 1917.

clearly distinguished. As to the accuracy of the information, especially in the sections on scientific method, my respect for distinguished professional colleagues prevents me from using such words as scandalous or disgraceful. I can only say that a blameless life as a minister of the gospel or as a college teacher does not qualify one, even after he has consulted previous text-books on logic, to become an authority on the methods of the various sciences—topics which only the masters of such sciences can justly deal with. My main point, however, is that the failure clearly to distinguish between the various points of view just mentioned has bred a great deal of the confusion of modern philosophy. I am not certain that logic can do much to train students in the habits of clear thinking. But logic ought certainly not to infect eager and trustful young minds with fundamental confusions, confusions which determine the setting of subsequent philosophizing. If this negative precept seem unimportant, I should like to remind you that modern hospital methods were revolutionized by Florence Nightingale, I believe, by this very insistence that, whatever else hospitals may do, they should not spread disease.

II. Nearly all the books define logic as in some manner the science of thought. But that the laws of logic are not the universal laws according to which we do actually think is conclusively shown, not only by the most elementary observation or introspection, but by the very existence of fallacies. Nor do we free ourselves completely from this difficulty by saying that logic deals with the laws according to which we think when we think correctly. Assuredly, correct thinking takes place only under favorable physiological, educational, and moral conditions. But we do not expect any treatise on logic to deal with the physiologic and moral conditions of mental health. To define logic as dealing with the laws according to which we ought to think does not define its distinctive subject, since the principles of every science are in a sense laws according to which we ought to think if we would think correctly on its distinctive subject-matter.

III. The distinctive subject-matter of logic, constituting, as a matter of fact, the core of the traditional Aristotelian logic, is what is called formal truth. The distinction between material and formal truth, like the related distinction between assumption and proof or between immediate and mediate truth, is not without its difficulties. But it is clear that we must distinguish between the factual truth of any proposition and the truth of the assertion that it logically follows from, or necessitates as consequences, certain other propositions. It is one thing to assert categorically that Nineveh fell in 622 B.C. and quite another to assert that, if it did, it must have pre-

ceded and can not have followed the Scythian invasion of 621 B.C. It is one thing to assert that through a point outside a straight line only one parallel can be drawn, but quite another matter to assert that, whether that proposition is true or not, from it and certain other propositions it necessarily follows that the sum of the angles of a triangle is equal to two right angles.

In any given context it is relatively easy to distinguish between the categoric assertion that a given proposition is true and the formal truth, validity, correctness or adequacy of the proof or demonstration that it follows from certain other propositions. In any given case, also, it is rather easy to see that the material truth of premises or conclusion and the validity of the proof may be relatively independent of each other—a proposition known to be false may be correctly proved (from false premises), and the proof of a true proposition may be formally defective. But when we come to deal with the general nature of formal truth and its relation to material truth we begin to encounter difficulties. Many, however, of the traditional difficulties may be eliminated if we take the trouble to distinguish clearly between reasoning or inference as an operation or event which happens in an individual mind and the question of evidence or general conditions under which what is asserted can be true. When this is recognized it becomes clear that logical or formal truths are truths concerning the implication, consistency, or necessary connection between *objects* asserted in propositions and the distinctive subject-matter of logic may be said to be the relations generally expressed by *if—then necessarily*.

Terms and relations, matter and form, immediate and mediate truth, are like north and south poles, strict correlatives, clearly distinguishable and inseparable—the existence of each is necessary to give meaning to the other. But obsessed by the monistic prejudice, philosophers have refused to recognize any ultimate polarity or duality,[2] and have perpetually sought to reduce everything either to form or to content. The former effort leads to empty panlogism, the latter to dumb mysticism. Without pretending to settle the problem, we may avoid the dilemma by recognizing that matter and form are strictly correlatives in every concrete situation, and that when we take the world of science as a whole it is found to contain besides logical relations an alogical element which no efforts of panlogism have successfully eliminated. At any rate there is no insuperable objection to the assertion that logical truth or consistency is a genuine part of the world of truth which science studies. What we call

[2] This difficulty of philosophers is precisely that of the exuberant individual who puzzled two policemen with the problem, "Which side of the street is *the* other side?"

the evidence for the material or factual truth of any proposition involves—excepting matters of immediate apprehension which are beyond argument—the question of the logical relation between the proposition in question and certain others which are taken for granted. Thus, we rule out the proposition that there is life on the planet Mercury, by the proposition that life can not exist except at certain temperatures, *etc.* In the same way the question of formal truth enters as an integral part of the material truth of what have been called practical propositions. Thus all of the practical judgments recently called to the attention of logicians by Professor Dewey can be put in the traditional hypothetical form of formal logic—if you want to recover, you must see a doctor, *etc.*[3] Indeed the relation of means to end is logically only one type of the relation of parts to wholes. This can readily be seen when we compare with the above practical judgments such theoretic judgments as, to complete the square you must add one to both members of the equation, or to reduce nine to seven subtract two, *etc.*

Similarly, formal consistency forms part of artistic or dramatic truth. If Hamlet is a prince, he must act consistently with the supposed nobility of that character. If he has studied many years at Wittenberg, he must show the scholar's aptitude for reflection on both sides of the question, to be or not to be.

It may be objected that in all these latter examples we are dealing with matters of fact that are contingent and not at all logically necessary. Life on this planet has been empirically found to exist between certain temperatures. In other planets it might be different. Even on this planet people may get well without consulting a doctor, princes may be undignified, and those who study at Wittenberg may not reflect at all. This objection, however, in no way militates against our account of the nature of the logical or formal relation. The objector raises an issue of fact. He challenges the material truth of the major premises assumed in these examples and not the necessity of the inferences drawn from them. This raises in each case an issue of fact to be settled by evidence, but it leaves unaffected the logical test of necessity which is, whether it is or is not possible for the antecedent to be true and the consequent to be false at the same time. In a world in which all princes are dignified it is impossible for Hamlet to act like a clown; and if the Hamlet on the stage does behave like a clown he only irritates us by his failure to live in the world of our assumption. When we demonstrate or prove a proposition in physics, *e. g.*, that if there were two bodies the smaller would on receiving a tangential motion

[3] It is because in practical judgments, the protasis or conditional clause is understood and not expressed that there arises the seeming difference.

describe an ellipse about the larger, we show that it is impossible for certain accepted principles (the law of gravitation) to be true and the demonstrated proposition to be false. When we come to the demonstrations of pure mathematics we do not restrict our postulate to any particular universe, but are concerned with the universe of all possible meaning. $2+2\neq 4$ is impossible, therefore, in any universe in which $2, +$ and 4 have the meanings assigned in our arithmetic.

IV. The need for considering the formal implications of an hypothesis, independently of the question whether it is in fact true, has led to the erroneous view that formal logic considers the consequences of propositions apart from their meaning. It ought to be clear, however, that a proposition devoid of all meaning would be just nonsense from which nothing could possibly be deduced. The particular logical consequences of any proposition surely do not follow from the mere sounds or marks on paper but from the nature of the objects asserted in the proposition. All scientific procedure, however, rests upon our ability to consider the abstract general characteristics possessed by all the objects of a group, leaving out of account the more specific nature in which they differ. Thus mechanics considers the mass and motion of bodies apart from their color, relative scarcity, or other property; and even more specialized sciences like crystallography, bio-chemistry or genetics, all consider isolated or abstract properties possessed by widely different objects. Similarly it is possible for logic to abstract from the specific concrete meaning of propositions those elements which are common to whole classes of propositions, and to denote these common elements by suitable symbols. When, therefore, mathematical logicians use such forms as *p implies q* they are not talking about propositions devoid of all meaning but about a certain property of classes of propositions.

The term property has a somewhat misleading connotation. It suggests an inert quality inhering in a substance. It may, therefore, be advisable to substitute the notion of operation or transformation for that of property.

In social usage, formal rules are rules of procedure applicable to all the members of a given class, irrespective of any personal characteristics such members may have. In the same way, every science has its rule of operation or laws according to which all the objects it studies can be combined. Logic is the most general of all the sciences; it deals with the elements or operations common to all of them. That is, rules of logic are the rules of operation or transformation according to which all possible objects, physical, psychical, neutral, or complexes can be combined. Thus, logic is an

exploration of the field of most general abstract possibility. This may make logical information very thin; but it is not therefore devoid of significance. Not only does it rule out impossibilities but it reveals the possibilities of hypotheses other than those usually taken for granted; and in this respect it frees the mind and contributes not only to the fixed form but to the living growth of science. The history of science shows beyond doubt that the vital factor in the growth of any science is not the Baconian passive observation but the active questioning of nature, which is furthered by the multiplication of hypotheses as hypotheses.

V. The foregoing explanation of what we mean by formal rules explains the great utility of symbols not only in logic and mathematics but in all exact sciences.

Whatever be the psychologic nature of the reasoning process, it is a fact that this process is facilitated by the use of artificial counters or symbols which represent only the general properties under investigation and not any of the specific properties which must be excluded. As the rules according to which our symbols can be combined are by hypothesis precisely those according to which the entities they denote can be combined, it follows that it is not necessary that we keep the concrete meaning or cash value of our counters always before us. If our reasoning is correct the meaning of our final result follows from our initial assumptions; and this, I take it, is one of the great advantages of any calculus or system of symbolic manipulation.

The employment of special symbols instead of the more familiar symbols called words, is a practical convenience rather than a logical necessity. There is not a proposition in logic or mathematics that can not be ultimately expressed in ordinary words (this is proved by the fact that these subjects can be taught to those who do not start with a knowledge of the special symbols). But practically it is impossible to make much progress in mathematics and logic without appropriate symbols, just as it is impossible to carry on modern trade without checks or book credits, or to build modern bridges without special tools. Symbolic reasoning is essentially reasoning on a large scale with instruments appropriate to such wholesale undertakings. If we want a large number of fish, we must use nets rather than single lines. The opposition to symbolic reasoning, like the old opposition to the introduction of machinery, arises from the natural disinclination to change, to incur trouble or expense for a future gain. The prejudice against careful analytic procedure is part of the human impatience with technique which arises from the fact that men are interested in results and would like to attain them without the painful toil which is the essence of our mortal finitude.

VI. The nature of the subject matter of logic may be better understood when it is seen to be identical with the subject matter of pure mathematics. This identity of logic and pure mathematics is the discovery of the nineteenth century, and was not possible before the discovery of non-Euclidean geometry and of multiple algebra revealed the true nature of pure mathematics. From the days of Plato to those of Kant, geometry was viewed exclusively as a science of physical space, and as the Euclidean axioms were regarded as self-evidently true, it was possible for Kant and his predecessors to maintain the existence of an *a priori* knowledge of nature. The discovery of non-Euclidean geometry shows that the axioms of the traditional geometry are convenient assumptions and not *a priori* necessities of thought or perception. Their contraries have been proved capable of receiving an equivalent logical and mathematical development, so that pure geometry alone is incapable of deciding the question of whether physical space is Euclidean or not. Geometry, as a branch of pure mathematics, serves only to develop the necessary consequences of various hypotheses or assumptions. Similar considerations apply to algebra, which used to be defined as the general science of number or quantity. The discovery of the real nature of the so-called imaginary numbers, and the consequent development of the various types of complex numbers and of various types of algebra, have brought out clearly that all algebra is essentially a calculus of the implication of certain rules of operation or combination. The commutative and associative laws of addition and multiplication are not necessities of thought, but assumptions which define specific transformations applicable only to those fields of nature to which they are empirically found to be applicable. But the rules or postulates of any algebra being laid down, the development is a matter of pure logic. Algebraic proofs are in every respect logical proofs and depend no more on any special element of intuition than does logic itself. For pedagogic or administrative purposes it may still be necessary to refrain from identifying mathematics with the whole region of necessary inferences in which all exact science is located, but in point of fact there is no significant difference between pure mathematics and deductive reasoning. What we usually call formal logic is simply the study of the most general portion of pure mathematics.

The assertion of the identity of logic and pure mathematics has appeared as a paradox and as a stone of stumbling to many philosophers, and even to some mathematicians. Surely, they tell us, a proposition about circles, quintic equations or prime numbers belongs to a different science than a proposition about syllogisms. This objection is perfectly valid so long as we uncritically accept

the views of mathematics and logic of our traditional school textbooks. If, *e. g.*, circles are viewed as objects in space, like stones or caterpillars, while logic deals with "laws of thought as such," then all talk about the identity of logic and geometry is sheer intellectual violence. But to take this view is to ignore the distinction between pure and applied mathematics. If we view circles as existing things in actual or physical space then geometry is a branch of physics or applied mathematics—the simplest branch of mechanics, as Newton has shown in the preface to the *Principia*. But geometry as a branch of pure mathematics is in no way concerned with the existence of circles in the physical world. Euclidean and non-Euclidean hypotheses can not simultaneously be true of the physical world, yet they are all equally legitimate branches of pure geometry, as is also the geometry of a four-dimensional space. Geometry, as a branch of pure mathematics, is interested in a problem of logical proof: whether if certain propositions (axioms, *etc.*) are true, certain other propositions must be so likewise. In the construction of its chain of demonstration, geometry, as has been shown by Pieri, Hilbert and others, does not need to use any concept except those definable in terms of the fundamental notions of logic (classes, relations, *etc.*), nor does it need to assume any primitive proposition except those assumed in logic. In pure geometry, then, propositions about points and lines are replaced by propositions about classes of indefinables and relations between them. You may object on linguistic ground, that propositions about classes and relations ought not to be called geometry, and that unless we continue to identify the indefinable "points" with the intuitable spots on paper or blackboard we ought not to keep the name geometry; but the significant fact remains that if you examine any rigorous treatise on plane geometry you will find that it will make no difference in the form and sequence of our propositions if our indefinable points are replaced by complex numbers, or if "distance between points" is replaced by differences of holiness in a multi-dimensional series of saints.

VII. A serious obstacle to the recognition of the identity of the subject-matter of logic and that of pure mathematics, an obstacle that has had a great influence on philosophers and mathematicians like Poincaré, is the assumption of the ancient dogma that in strict deduction there can be nothing in the conclusion which is not already contained in the premises. From this it is argued that mathematics, so fertile in unexpected discoveries, can not be purely deductive. Any argument that a certain thing can not be is refuted if we can actually show it, and to the contention that mathematics can not be reduced to formal logic, the actual doing of it by Frege, Peano, Pieri, and Whitehead and Russell is sufficient refutation. It is in-

structive, however, to examine the dogma at the basis of this obstinate refusal to admit an established fact, especially since the dogma is closely related to the generally accepted but essentially obscure dictum that all knowledge comes from experience.

The notion that deductive reasoning must necessarily be a sterile series of tautologies arises from the failure to distinguish between psychologic, physical, and logical considerations. Psychologically it is obviously not true that the conclusion is always contained in the premises. For ages men accepted the elementary laws of arithmetic without seeing that they involve as a necessary consequence the proposition that there are no two numbers whose ratio is the square root of two. Or, to take a more concrete example, I may know that the *Camperdown* was sunk and none aboard could be saved, and I may know also that Smith sailed aboard that ill-fated vessel. And yet it may be some time before the union of these two propositions flashes on my mind the startling conclusion that Smith must have been drowned.[4] To suppose that when we think of any proposition or group of propositions, we always have in mind all their logical consequences is a supposal inconsistent with the fact that many find the study of mathematics difficult or are easily tripped by lawyers on cross-examination. Nor is strict deduction incompatible with the existence of physical novelty, *i. e.*, with the coming into being at certain moments of time of that which did not exist at previous moments of time. The fact that the moon is every moment in a new position does not make it impossible to deduce a comprehensive formula for its path out of a few past observations and the hypothesis of universal gravity.

The consequences in a deductive system, then, may be new in time as well as psychologically startling or unexpected, and yet there will be no proposition in our series which is not necessitated by the premises. The difficulty with the traditional doctrine arises from the prevailing confusion between the process of thinking or learning which takes place in time, and the logical relations discovered, which do not form a temporal series at all. *In natura rerum* premises do not exist prior to their conclusion any more than they exist to the right or to the left of them. The spatial and temporal order is of very wide application, but we must guard against its undue extension. Thus it is well to note that when we speak of the con-

[4] The silly character of the old argument that every syllogism involves a *petitio principi* because no universal can be known before we know all the particulars under it, becomes clear when we take a practical syllogism such as, all persons convicted of crime should be disfranchised, my brother has been convicted, *etc.* Jephthah said, Whoever cometh forth, *etc.* Yet he was surprised when his daughter proved to be the one.

clusion being contained in the premises, we are resorting to an uncritical spatial metaphor. The inexhaustible theorems of algebra are assuredly not contained in its few axioms or primitive propositions in the way in which the chairs and other objects are contained in a room. All the possible games of chess that can be played can be deduced from the few rules of that game. But the games are not literally contained in these rules. The notion of containing may, indeed, be used in a wider sense to denote a certain relation of order, of which the spatial relation of container and contained is one instance. But in this wider sense not only are the games contained in the rules but the rules contained in the multitude of games, as invariant changes or transformations common to all of them. The particular is in one sense part of the universal but in another sense the universal is simply that part or aspect of the particular which is the object of study. The tremendous usefulness of general propositions and the predilection of Greek rationalism and medieval authoritarianism have spread the view that general truths have something of a superior status, superior certainty, superior authority and what not. But in respect of logic, premises and conclusions are on the democratic basis of strict correlatives. Logic shows that certain premises are sufficient or necessary for certain conclusions or that certain conclusions necessarily follow from certain premises. *The categoric assertion of either premises or conclusions involves something more than logic.* If, then, the laws of logic are rules of combination, nothing can be deduced from them except various combinations of logical rules. And it is as impossible to derive physical or psychologic truth from pure logic as to build a house with nothing except the rules of architecture. To say, however, that there is nothing in any logically or mathematically developed science except what is contained in its data is to say that there is nothing in a building except what is contained in its bricks, mortar, and other materials. The form or structure of a house is constituted by the system of relations between the material entities which make it up; and the form or structure which logic studies is the system of relations which hold between all possible objects that can be ordered into a system.

VIII. According to the prevailing view, the relations between premises and conclusion exist in the mind only. This means either that terms and propositions apart from their relations exist in the so-called external world, or else that nothing at all exists outside of the mind. If we put terms in one world and relations in another, it is difficult to see how the terms can have an intelligible or knowable character, and how relations in one universe can be said to be the relations of terms in another. This is the basis of the familiar but unanswerable difficulties of epistemology—how ideas in a mind

can know things in a world external to it. If, on the other hand, the terms logically related also exist in the mind, then the distinction between logic and physics is still to be maintained, and the distinction between the two can not be derived from their common mental nature. One engaged in an actual logical or mathematical investigation can no more make any progress towards a definite solution of a problem by invoking any doctrine as to the nature of mind or, thought than he can by invoking a theory as to the nature of God and His providence.

IX. It has always been recognized that logic deals with relations that are necessary, but the nature of necessary relations has been obscured, first, by the Stoic confusion between that which is necessary and that which is generally accepted, and by the modern identification of necessity with psychologic certainty. Obviously, the existence of fallacies proves that we may be certain of many things which are not necessarily true, and the widespread extent of such certainty is not of itself a logical proof—at least, not in the field of an exact science like mathematics or physics. Perhaps it is the subservience of logic to rhetoric (the art of disputation) that has caused us to look upon logical proof exclusively as a method of producing certainty or conviction. The essence of deduction or proof, however, is not the psychologic certainty which it may or may not produce, but the exhibition or demonstration of the logical structure of the system studied. The fact that a theorem about the sum of two sides of a triangle being greater than the third is derived from a Euclidean axiom does not add to its psychologic certainty; but it does reveal the structure of the Euclidean system in showing that in so far as that theorem is concerned no additional axiom is necessary.

Certainty is a primal need of the intellectual life. We all need some ground from which to start and on which to light after our short swallow flights of doubt and critical reflection. Some walk with firmer foot in answering the question: what facts exist? and some in answering the question: what claims are valid? Logically, however, existence and validity are strictly correlative. We must admit certain things to exist because their claims are valid, and claims are valid because they exist as such. The existence of the logical or relational structure of Euclidean geometry is as much a fact as the composition of albumen, the structure of rocks, or the constitution of the solar system, all of which depend on geometric relations. But if the distinction between logic and physics as indicated above is valid, it is well to distinguish between logical relations which are necessary, and factual relations which are contingent. This distinction may be made in two ways: First, particular sciences

like physics may be said to start with material assumptions; *i. e.*, assumptions true only of certain objects, namely, entities occupying time and space, while logic assumes only laws applicable to all objects. The second way of drawing the distinction is to say that while physics and other special sciences assume systems governed by laws the contraries of which are abstractly possible, logic deals with laws whose contraries are devoid of meaning or application to any possible determinate objects. The assumptions of even the most developed physical science, such as mechanics, can be shown to be sufficient, but can not be proved necessary, since it is possible that some other hypothesis may explain the facts. But the assumption that the objects of physics and other sciences must conform to logic is necessary in the sense that without it no science at all can be constructed.

X. Against the view that logic explores the realm of the possible and the necessary we have the extreme nominalism or empiricism of men like Hume, Mach, and Schiller, who deny the existence of objective necessary relations and reduce everything to a consideration of the actual existence of terms or impressions. This glorification of the category of existence and disparagement of the categories of possibility and necessity shows itself in its clearest form in Mach's contention that the world is given but once and that it is not valid to argue as to what would have happened if things were different. Mr. Schiller is a loyal disciple of Mr. Bradley in his distrust of abstractions. Mr. Brunschvicg, in a recent book, thinks it a triumphant argument against the new logical realism that it is as applicable to the world of Poe's imagination as to the real world of science. The error underlying this view is as profound as it is widespread. The category of reality belongs not to science but to religion. It arises not as an aid to an intellectual analysis of our world, but as a means of escape or deliverance from the perplexities and confusions of deceitful appearances in a disorderly world. At any rate, it is rather easy to show that the prejudice in favor of reality (and the special form of it which glorifies the category of existence) is based on an inadequate analysis of the nature of science. Science would be impossible if we could not study the consequences of materially false hypotheses. In all sciences the consequences of rival hypotheses, such as those concerning the ether, must be deduced irrespective of their material truth, and indeed as a necessary condition before the material truth can be determined. Though two contradictory hypotheses can not both be true in the material or existential sense, both must be assumed to have determinate consequences. The realm of science can not, therefore, be restricted to the realm of actual or historic existence. Indeed, determinate exist-

ence without any reference to possibility would be meaningless in mundane affairs. Science studies the character or determinate properties of things, whether actual or possible. In this respect science, like art and practical effort, liberates us from the prison-house of the actual and enables us to penetrate beyond to the region of the possible. What we call ideals or hypotheses are our guides in the labyrinth of possibility. The positivists who boast that they are concerned only with what is, like the hard-hearted statesmen or business men who say that they deal only with hard actualities, are deluding themselves with fantastical dogmas, hiding the crudity of their ideals with the pretense that they have none.

XI. We have used the term logic or formal logic as identical with deduction. But as modern text-books on logic devote more and more attention to what they call methodology and induction, a few remarks on these subjects are called for.

Though the term "method" is one of the most frequently used, it is one of the least frequently defined terms in the whole repertory of philosophy. It is, therefore, best to examine what is actually treated under the head of methodology, and this I think will always be found to fall under one of the following heads:

1. Elementary ideas or general principles, culled from the various special sciences, and stated perhaps in a more abstract and uniform language than in the books professing to deal with these special sciences directly (*e. g.*, Bain's *Logic*).

2. An account of the psychologic processes involved in scientific thought, *i. e.*, in the process of learning or scientific demonstration (*e. g.*, Sigwart and his followers).

3. Historical information as to the way certain great scientific discoveries are supposed to have been made, and

4. Directions as to how science is to be cultivated so as to lead to discovery of laws or causes.

Of these the first three may be auxiliary to pure logic but certainly outside of its proper domain, while the fourth is entirely beyond its competence.

Though the idea of logic as an organon or aid in discovery seems to be as ancient as the science of logic itself, it does not seem to me that this claim can be seriously supported on behalf of either the ancient Aristotelian or the modern Baconian logic. In the main it is true enough that a knowledge of truths already known is the principal condition for the discovery of new ones, and the knowledge of any science may thus indirectly help in the exploration of any other field; but the science which will teach every one to become a discoverer of new laws has not yet been found, and the student of logic as such seems (if we judge by past experience) to be the least likely

to find it. The logician can not pretend to be able to act as intellectual physician or trainer to the scientific specialist. He may at best, if he takes the unusual trouble of familiarizing himself thoroughly with the subject-matter of the various sciences, act a part similar to that of the analytic critic of literature—he may indicate significant identities and differences in the various sciences and criticise the adequacy of the evidence for certain general contentions. Such a comparative logical study would, if developed, be of inestimable value, but it would belong to applied rather than pure logic.

On the subject of induction I can but repeat the statement made some years ago by Bertrand Russell, that all inference is deductive and that what passes as induction is either disguised deduction or more or less methodical guesswork.

This statement has shocked many logicians who do not like to admit that in science as in other fields of life guessing can play a part. But it is to be noted that the fact that a proposition is arrived at by a process of guessing does not determine its truth or falsity nor the purely logical question of its relation to other propositions. It is therefore absurd to draw a sharp antithesis between induction as a method of discovery and deduction as a method of exposition. Deductive logic and pure mathematics generally deal with certain relations between propositions, and the knowledge of such relations is certainly one of the most potent instruments of scientific research.

A brief glance at some typical views of induction may perhaps make my meaning clearer.

The term induction has been used to denote among others:

1. Reasoning from facts or particulars to laws or universals (Boethius and the scholastics).
2. Reasoning which is based on the principle of uniformity of nature *i. e.*, like effects must have like causes (Mill), and
3. Disjunctive reasoning (Schuppe, Montague).[5]

1*a*. As to the first, we must start with the observation that science does not know or does not deal with absolute particulars or pure facts—at least it never draws any inference from any sense-data except when the latter are viewed as already embodying or illustrating certain universals. It would obviously be impossible to state what happened in a single laboratory experiment except in terms of abstract or universal properties, such as weight, velocity, change, *etc.* There is, therefore, in fact no such thing as reasoning from pure particulars.

1*b*. If the sharp metaphysical separation between facts and laws is waived, and induction is defined as reasoning in which the end or

[5] Schuppe, *Erkenntnistheorie,* pages 53 ff.; Montague, *On the Nature of Induction,* this JOURNAL, Vol. III., pages 281 ff.

conclusion is more general than the beginning or premises, this account of the matter is still untenable. In deductive geometry or algebra we can pass by strict deductive steps from the nature of triangles (*e. g.*, in respect to area) to that of all polygons, or from the nature of integers to that of numbers generally (as in the binomial theorem).

2. Mill's account of induction makes it synonymous with reasoning by analogy. "This medicine cured my little girl, therefore it will cure yours." Now there can be no doubt that this is the way most people actually reason, in the sense that this is what they are conscious of as what goes on in their minds. But in truth the consequence follows from the premise only when your little girl is like mine in all respects in which the given medicine is applicable. Hence, as scientific medicine develops, the question whether their cases are alike comes to the foreground and the argument changes from a blind empiricism to an argument which tends to assume the explicitly deductive form.

3. The account of induction which makes it synonymous with disjunctive reasoning seems to me thoroughly sound and illuminating. In actual scientific inquiry we start with a number of merely or barely possible explanations. The cause of A may be C, D or E, or any other number of circumstances. If one of these hypotheses be true certain consequences should follow, and any failure of one of these consequences rules out the hypothesis and thus diminishes the number of alternatives. This explains how it may happen that a single experiment may lead to the elimination of all but one possibility and therefore the definitive establishment of a law.[6] From this point of view Mill's method of agreement and difference[7] has a limited usefulness as a method of eliminating the circumstances which are not causal, and thereby helping somewhat in finding the true cause. But it is to be observed that the efficiency of this method depends on our fundamental assumption as to what circumstances are relevant or possibly related causally to the given effect. If the true cause is not included in our major premise the "canons of induction" will not enable us to discover it. If any one thinks that I have understated the case for these canons of induction as methods of discovery, let him discover by their means the cause of cancer or of disorders in internal secretions.

XII. To sum up the position of this paper: The field of every science consists of the relations of certain constants and variables. The

[6] This is the *schema* of a crucial experiment. In the actual history of science things are more complicated, and none of the historical instances of crucial experiments given in the logic books were in fact as decisive as the books pretend.

[7] The method of residues is simply the disjunctive syllogism over again.

constants need not be enduring substances but may be the invariant laws according to which the changes take place. If the actually formulated laws of our physics can be shown to undergo change themselves, it can only be in reference to something else which is constant in relation to them. This justifies the Kantian contention for *a priori* elements in experience, in the sense that every science must assume some invariant connections or categories. The Kantians, however, are wrong in claiming absolute logical necessity for material principles such as those of Euclid's geometry, Newton's mechanics or Christian ethics. These principles are assumptions which may be necessary for some of the consequences drawn from them, but they are not absolutely necessary, since it is possible to reject these consequences. This view agrees with the experimental theory of knowledge and morals, except so far as the latter seems to repeat Hume's denial of objectively necessary relations or rules. Without the latter there can be no rational experiment or significant doubt.

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PROFESSOR SPAULDING'S NON-EXISTENT ILLUSIONS

PROFESSOR SPAULDING'S recent volume entitled *The New Rationalism* must be a source of delight to every realist of whatever school. Seldom has so thorough-going a defense of realism been made. Relatively little emphasis, moreover, has been put upon those special features which characterize that school of realism to which Professor Spaulding has for years belonged, and in fact by the omission of a few passages a dualistic realist might perfectly well accept all of the author's arguments and conclusions. The one point of importance upon which his views diverge greatly from those of other realists who do not care to call themselves "new" is to be found in his retention of "pan-objectivism" and his insistence that illusion, hallucination, and error must not be classed as mental. If in examining this part of Professor Spaulding's position I seem to be severe in my criticisms, I trust he will remember that I am prompted thereto by my sympathy with and my admiration for the greater part of his admirable book and by the conviction that he is nearer to the old and true type of realism than he is himself aware.

The question of illusion and error is touched upon, in various parts of *The New Rationalism,* but in no place is there an inclusive and systematic statement of the author's position upon the subject.